Math 10B with Professor Stankova
Worksheet, Discussion #25; Thursday, 4/25/2019
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T-Test

Concepts

1. You use a T test if your sample size is small $n < 30$. The **t-statistic** for a value x is $\frac{|x-\mu|}{\sigma/\sqrt{n}}$, the same value we calculated for the z test. The **degrees of freedom** is $\nu = n-1$. The PDF of the t distribution with ν degrees of freedom is

$$f_\nu(x) = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}.$$

The **Γ-function** is $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$. It satisfies the property that $\Gamma(x+1) = x\Gamma(x)$ for all $x > 0$, $\Gamma(n) = (n-1)!$ for $n \geq 1$, and $\Gamma(1) = 1$, $\Gamma(1/2) = \sqrt{\pi}$.

Examples

2. An infomercial claims that a miracle drug will cause you to grow all your hair back. There are 25 brave participants and surprisingly 7 people regrew their hair. If normally 10% of people regrow their hair, can you say that this drug worked?

Problems

3. True False As the number of degrees of freedom increase the probability that $P(T \geq t) \to 0$ for fixed t.

4. True False For large degrees of freedom, the values gotten from the t table are the same as those from a z score table.

5. True False The PDF of the t distribution is only defined for positive values of x.

6. True False It is possible to do a t test with only one measurement.

7. You think that legacy students are being admitted to college with lower test scores than non-legacy students. Suppose that the average SAT score of non-legacy students is $\mu = 1300$. You ask 9 legacy students what their SAT scores were and get a sample mean of $\bar{x} = 1270$ and a sample standard deviation of $s = 90$. Can you say that legacy students are being admitted with lower standards with $\alpha = 10\%$?

8. You think that legacy students are different from non-legacy students. Suppose that the average SAT score of non-legacy students is $\mu = 1300$. You ask 9 legacy students what their SAT scores were and get a sample mean of $\bar{x} = 1350$ and a sample standard deviation of $s = 75$. Can you say that legacy students are different from non-legacy students with $\alpha = 0.05$?

9. The heart rates of 4 patients in an ICU have mean 96 beats per minute and standard deviation 16. Are the heart rates for ICU patients unusual given that the normal heart rate has a mean of 72 beats per minutes with $\alpha = 0.05$?

10. Calculate $\Gamma(4)$ and $\Gamma(5/2)$.

11. Write the PDF for the t distribution when $\nu = 4$.

12. Prove that $\Gamma(x+1) = x\Gamma(x)$ for all $x > 0$.

13. Use induction to prove that $\Gamma(n) = (n-1)!$ for all $n \geq 1$.

Goodness of Fit Testing

14. You use a χ^2 test to determine if a distribution is how you expect it to be. Suppose that you expect it to be distributed with a different values and for each of these values, you expect to get outcome k m_k times but actually get it n_k times. Then you compare the statistic
$$r = \sum_{k=1}^{a} \frac{(n_k - m_k)^2}{m_k} = \sum \frac{(\text{observed - expected})^2}{\text{expected}}$$
with the $\chi^2_{k=a-1}$ distribution. The χ^2 distribution with $k = a-1$ degrees of freedom has the PDF for $x \geq 0$
$$f(x) = \frac{1}{2^{k/2}\Gamma(k/2)} x^{k/2-1} e^{-x/2}.$$

Example

15. In a skittle bag, you get 11 red skittles, 12 blue, 5 green, 10 yellow, and 13 orange skittles. Is it possible that the colors are evenly distributed with a significance level of $\alpha = 0.05$?

Problems

16. True False If the critical value for the χ^2 distribution with k degrees of freedom is r, then $P(R \geq r) = \alpha$.

17. True False For fixed significance level α, as the number of degrees of freedom increases, the critical value also increases.

18. You take 400 cards and get 100 spades, 105 hearts, 107 diamonds, and 88 clubs. Can you say that the suits are not evenly distributed with $\alpha = 0.05$?

19. You expect to get a distribution of brown eyes brown hair to brown eyes blond hair to blue eyes brown hair to blue eyes blond hair as $9 : 3 : 3 : 1$. When looking around in class, you get a distribution of $61 : 19 : 11 : 9$ after looking at 100 people. Is this distribution accurate (use $\alpha = 0.05$)?

20. Find $\chi^2(x)$ for $k = 1, 2, 4$.

21. Use induction to prove that $E[\chi^2_{k=2n}(x)] = 2n$ for all $n \geq 1$.

$$\textbf{p-values for one-tailed } t\textbf{-distribution} = \int_{\mathbf{t}}^{\infty} \mathbf{f}_\nu(\mathbf{x})\mathbf{dx}$$

$\nu = n - 1$ degrees of freedom $\qquad t = t$-score

$t \downarrow \quad \nu \rightarrow$	1	2	3	4	5	6	7	8	9	10
0.0	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.2	0.437	0.430	0.427	0.426	0.425	0.424	0.424	0.423	0.423	0.423
0.4	0.379	0.364	0.358	0.355	0.353	0.352	0.351	0.350	0.349	0.349
0.6	0.328	0.305	0.295	0.290	0.287	0.285	0.284	0.283	0.282	0.281
0.8	0.285	0.254	0.241	0.234	0.230	0.227	0.225	0.223	0.222	0.221
1.0	0.250	0.211	0.196	0.187	0.182	0.178	0.175	0.173	0.172	0.170
1.2	0.221	0.177	0.158	0.148	0.142	0.138	0.135	0.132	0.130	0.129
1.4	0.197	0.148	0.128	0.117	0.110	0.106	0.102	0.100	0.098	0.096
1.6	0.178	0.125	0.104	0.092	0.085	0.080	0.077	0.074	0.072	0.070
1.8	0.161	0.107	0.085	0.073	0.066	0.061	0.057	0.055	0.053	0.051
2.0	0.148	0.092	0.070	0.058	0.051	0.046	0.043	0.040	0.038	0.037
2.2	0.136	0.079	0.058	0.046	0.040	0.035	0.032	0.029	0.028	0.026
2.4	0.126	0.069	0.048	0.037	0.031	0.027	0.024	0.022	0.020	0.019
2.6	0.117	0.061	0.040	0.030	0.024	0.020	0.018	0.016	0.014	0.013
2.8	0.109	0.054	0.034	0.024	0.019	0.016	0.013	0.012	0.010	0.009
3.0	0.102	0.048	0.029	0.020	0.015	0.012	0.010	0.009	0.007	0.007
3.2	0.096	0.043	0.025	0.016	0.012	0.009	0.008	0.006	0.005	0.005
3.4	0.091	0.038	0.021	0.014	0.010	0.007	0.006	0.005	0.004	0.003
3.6	0.086	0.035	0.018	0.011	0.008	0.006	0.004	0.003	0.003	0.002
3.8	0.082	0.031	0.016	0.010	0.006	0.004	0.003	0.003	0.002	0.002
4.0	0.078	0.029	0.014	0.008	0.005	0.004	0.003	0.002	0.002	0.001
4.2	0.074	0.026	0.012	0.007	0.004	0.003	0.002	0.001	0.001	0.001
4.4	0.071	0.024	0.011	0.006	0.004	0.002	0.002	0.001	0.001	0.001
4.6	0.068	0.022	0.010	0.005	0.003	0.002	0.001	0.001	0.001	0.000
4.8	0.065	0.020	0.009	0.004	0.002	0.002	0.001	0.001	0.000	0.000
5.0	0.063	0.019	0.008	0.004	0.002	0.001	0.001	0.001	0.000	0.000
5.2	0.060	0.018	0.007	0.003	0.002	0.001	0.001	0.000	0.000	0.000
5.4	0.058	0.016	0.006	0.003	0.001	0.001	0.001	0.000	0.000	0.000
5.6	0.056	0.015	0.006	0.002	0.001	0.001	0.000	0.000	0.000	0.000
5.8	0.054	0.014	0.005	0.002	0.001	0.001	0.000	0.000	0.000	0.000
6.0	0.053	0.013	0.005	0.002	0.001	0.000	0.000	0.000	0.000	0.000
6.2	0.051	0.013	0.004	0.002	0.001	0.000	0.000	0.000	0.000	0.000
6.4	0.049	0.012	0.004	0.002	0.001	0.000	0.000	0.000	0.000	0.000
6.6	0.048	0.011	0.004	0.001	0.001	0.000	0.000	0.000	0.000	0.000
6.8	0.046	0.010	0.003	0.001	0.001	0.000	0.000	0.000	0.000	0.000
7.0	0.045	0.010	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000
7.2	0.044	0.009	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000
7.4	0.043	0.009	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000
7.6	0.042	0.008	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000
7.8	0.041	0.008	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000
8.0	0.040	0.008	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000
8.2	0.039	0.007	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000
8.4	0.038	0.007	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000
8.6	0.037	0.007	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000
8.8	0.036	0.006	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.0	0.035	0.006	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.2	0.034	0.006	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.4	0.034	0.006	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.6	0.033	0.005	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9.8	0.032	0.005	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.032	0.005	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Chi-square Distribution Table

d.f.	.995	.99	.975	.95	.9	.1	.05	.025	.01
1	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	6.63
2	0.01	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21
3	0.07	0.11	0.22	0.35	0.58	6.25	7.81	9.35	11.34
4	0.21	0.30	0.48	0.71	1.06	7.78	9.49	11.14	13.28
5	0.41	0.55	0.83	1.15	1.61	9.24	11.07	12.83	15.09
6	0.68	0.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81
7	0.99	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89
32	15.13	16.36	18.29	20.07	22.27	42.58	46.19	49.48	53.49
34	16.50	17.79	19.81	21.66	23.95	44.90	48.60	51.97	56.06
38	19.29	20.69	22.88	24.88	27.34	49.51	53.38	56.90	61.16
42	22.14	23.65	26.00	28.14	30.77	54.09	58.12	61.78	66.21
46	25.04	26.66	29.16	31.44	34.22	58.64	62.83	66.62	71.20
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15
55	31.73	33.57	36.40	38.96	42.06	68.80	73.31	77.38	82.29
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38
65	39.38	41.44	44.60	47.45	50.88	79.97	84.82	89.18	94.42
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.43
75	47.21	49.48	52.94	56.05	59.79	91.06	96.22	100.84	106.39
80	51.17	53.54	57.15	60.39	64.28	96.58	101.88	106.63	112.33
85	55.17	57.63	61.39	64.75	68.78	102.08	107.52	112.39	118.24
90	59.20	61.75	65.65	69.13	73.29	107.57	113.15	118.14	124.12
95	63.25	65.90	69.92	73.52	77.82	113.04	118.75	123.86	129.97
100	67.33	70.06	74.22	77.93	82.36	118.50	124.34	129.56	135.81